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Creativity and scientific discovery with infused design and its analysis with C–K theory

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Abstract Creativity is central to human activity and is a powerful force in personal and organizational success. Approaches to supporting creativity are diverse and numerous. The only way to understand the diversity and utility of these methods is through their careful analysis. The analysis conducted in this paper is done with the aid of a theory. As a first step, we use infused design (ID) method to generate new concepts and methods in the classic discipline of statics, in addition to its prior use in the generation of a number of creative designs. The use of the ID method in the creative scientific discovery process is modeled with C–K design theory, leading to better understanding of ID and C–K. The exercise in this paper illustrates how the synthesis of a theory, a framework, and methods that support discovery and design is useful in modeling and evaluation of creativity methods. Several topics for future research are described in the discussion.

Keywords Design theory · Creativity · Infused design · C–K theory · Scientific discovery

An early and shorter version of this paper was presented in ICED09 and won the outstanding paper award (Shai et al. 2009b).

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1 Introduction

Creativity is central to human activity and is a powerful force in personal and organizational success. As the interest in the subject never ceases to grow, new methods for enhancing creativity are constantly proposed. The only way to understand the diversity and utility of these methods is through their careful analysis.

In several interrelated studies, we initiated our efforts towards systematic analysis of creativity methods by defining a general framework that organizes the methods and illustrating the analysis by comparing specific methods within a formalization of a design theory (Reich et al. 2008; Shai et al. 2009a; Reich et al. 2012). The present study continues that thrust by showing how new concepts and theorems in engineering could be derived by using infused design (ID), resulting in a creative act that is also considered scientific discovery. ID is a design method that supports the transfer of knowledge between disciplines and through this, the ability for creative design (Shai and Reich 2004a, b; Shai et al. 2009a). Later in this paper, we extend our exploration by showing how the creative act that is supported by ID is describable within C–K theory—a formal design theory that embeds creativity as a central part of its scope (Hatchuel and Weil 2003, 2007, 2009).

Specifically, this paper shows the process of using ID for generating new entities or variables and theorems in the classic field of statics. A discovery of such entities and theorems would be considered as a high level of creative thinking.[1] This creative act follows a heuristic that has

[1] The A.T. Yang Memorial Award in Theoretical Kinematics was awarded to the discovery described in this paper at the *29th Biennial Mechanisms and Robotics Conference* in Long Beach, CA, September 2005.

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been fruitful in science before, which could be referred to as "filling the holes in the map." A famous example of its use in science is related to creating the periodic table by Dmitri Mendeleev in 1869 and its refinement into a model of the atoms with missing elements. Mendeleev table was better than another contemporary table of its time in that it left gaps for elements such as thorium with the prediction of their properties derived from the underlying structure of the table. Mendeleev was so sure of the underlying logic of the table that "he was prepared to question the experimentally determined atomic weights in cases (such as the element thorium) where they seemed to conflict with his ordering scheme. (Ball 2002, p. 105)" Through the years, these missing elements attracted attention that led to their subsequent discovery.

In this paper, ID provides the mechanism to fill gaps located in holes identified in a representation of multidisciplinary knowledge called interdisciplinary engineering knowledge genome (IEKG, Reich and Shai 2012). The precise steps are detailed later in the paper, and for now, it is interesting to say that the process in ID described in this paper engages two types of dualities: the graph theory duality and the projective geometry duality. The modeling of the complete process in C–K has led to better understanding of ID and C–K. The exercise in this paper illustrates how the synthesis of a framework and methods that support discovery and design with a design theory is useful in modeling and evaluation of creativity methods. As such, other studies that evaluate creativity methods by a list of properties are challenged to provide also a theory-based evaluation.

The remainder of this paper is organized as follows. Section 2 reviews the methodology of our studies. Section 3 provides a brief overview of ID and C–K. Section 4 presents a case study in which ID was used to create new concepts in a discipline that is so traditional and accomplished, that it would seem unlikely that such a new concept could have been discovered. The precise articulation of this discovery is a contribution in itself. Section 5 is the core of the analysis, explaining the case study in C–K terminology and Sect. 6 concludes the paper.

2 Methodology

Shai et al. (2009a) described a conceptual analysis of creative conceptual design methods. The analysis allows incorporating issues that escape formalisms such as the cognitive style of designers or the design culture of the firm. As such, the analysis is qualitative, leading to classification of methods and potentially induced correlations between some method aspects.

Reich et al. (2012) described a methodology for conducting theory-based studies of creativity. In this methodology, methods are analyzed with respect to design theories. The first effort in theory-based analysis modeled a family of similar creativity assisting methods (ASIT and partially TRIZ, SIT, and USIT, which all work by using various types of templates) within C–K theory. This analysis led to several insights including: ASIT provides a specific method to realize C–K operators; C–K theory captures ASIT fully; and C–K theory provides insights to extend ASIT. A theory-based analysis is more specific and could lead to detailed insight as opposed to classification.

The analysis presented in the current paper applies the same methodology to illustrate the relationship between ID and C–K theory. But there are differences between the two analyses originating from the particular methods analyzed. In the ASIT case, the analysis concentrated on a single source of knowledge that is used to generate patterns for expanding C-concepts in the C-space of C–K theory. In the present analysis, ID is used to bridge the gap between different disciplines. Therefore, it offers insight about what might be done to facilitate knowledge transformation within the K-space of C–K theory. As we shall see in the next sections, the analysis reveals new insights about C–K and ID.

Altogether, while different, the two methodological studies: the conceptual (Shai et al. 2009a) and the theoretical (Reich et al. 2012; the present paper) complement each other.

3 Brief review of C–K and ID

3.1 Infused design

Infused design (ID) is a method that rests on a solid mathematical foundation for combinatorial representations of systems called the interdisciplinary engineering knowledge genome (IEKG; Reich and Shai 2012). It is a collection of discrete mathematical representations interconnected by mathematical relations among themselves that can model diverse engineering disciplines (Fig. 1). ID has demonstrated the ability to generate new forms of creative designs that were not conceived before, by studying and transferring across disciplines designs from seemingly unrelated disciplines (Shai et al. 2009a; Shai 2005a, b).

The representations that are the foundation of ID are discrete mathematical models, called graph representations; they include resistance graph representation (RGR), potential graph representation (PGR), flow graph representation (FGR), and others. These representations can represent diverse systems, for example, RGR is isomorphic representation of both electrical circuits and indeterminate trusses (Shai 2001b). These representations and their relations (see Fig. 1), such as the duality between PGR and

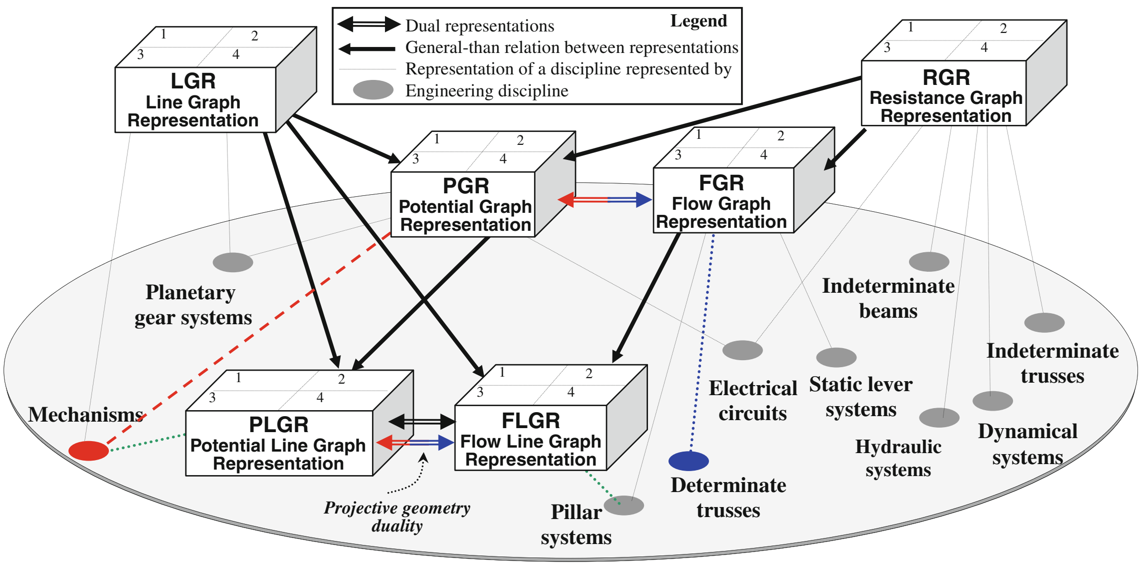

Fig. 1 Map of graph representations, their interrelations, and association with engineering systems (Shai and Reich 2004b). The map includes the relationships between mechanisms and determinate trusses that are used in this paper

FGR allow for transforming automatically one representation to others connected to it (Shai 2001a; Shai et al. 2009a). The automated transformation in ID is provably mathematically correct as these transformations are guaranteed to produce the same behavior for the original and transformed representation. This is in contrast to other creativity assisting methods, such as analogy, that do not guarantee that the transformation process across disciplines will lead to the preservation of the behavior of the original representation and consequently include steps for evaluation, repair as well as abandoning the analogy (Hall 1989).

We have discovered that one could organize discrete mathematical models in a hierarchical order from simple models describing systems with basic generic functionality to more compound models describing complex systems. We call the simple models—“systems genes” as they can be transformed into actual systems in different disciplines by traversing through the network of representations. In addition, we can also identify “method genes” as basic methods that operate on graphs that can be transformed into methods that transcend different disciplines. One such example is the cutset method that could be transformed into displacement method in structures and node method in electrical circuits. There are other types of genes that could be identified in the graph representation such as “structural system genes” (Shai and Reich 2011). The collection of genes and their interrelations is called IEKG (Reich and Shai 2012).

To better illustrate the process of ID, consider an example where all the disciplines that participate in the design are modeled in Fig. 1. A discipline that is still not represented cannot participate in the process. Members of the multidisciplinary team start by using their customary disciplinary model and terminology for each discipline, for example, PGR for mechanisms and FGR for static systems. In order to integrate all the disciplinary representations, they need to traverse the map of representations to find one common representation that accommodates all the original representations. For this particular example, according to Fig. 1, PGR, FGR, and RGR could serve as the common representation because PGR and FGR are dual and because RGR is more general to both.

Once the common representation is found, there is a path in the representations map that allows for transferring knowledge from one discipline to the other. This knowledge includes solutions or solution methods.

3.2 C–K theory

C–K theory, at the core of its scope explains creative thinking and innovation. It makes use of two spaces: (1) *K*—the knowledge space—is a space of propositions that have a logical status for a designer; and (2) *C*—the concepts space—is a space containing concepts that are propositions, or groups of propositions that have no logical status (i.e., are undecidable propositions) in *K*. This means that when a concept is formulated, it is impossible to prove that it is a proposition in *K*. Design is defined as a process that generates concepts from an existing concept or encodes a concept into knowledge, that is, propositions in *K*.

Concepts can only be partitioned or included, not searched or explored in the C-space. If we add new properties ($K \rightarrow C$) to a concept, we partition the set into subsets; if we subtract properties, we include the set in a set that contains it. No other operation is permitted. After

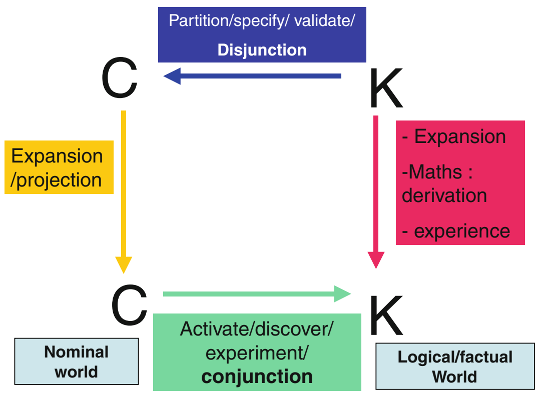

Fig. 2 The design *square modeled* by C–K theory (Hatchuel and Weil 2003)

partitioning or inclusion, concepts may still remain concepts ($C \rightarrow C$) or can lead to the creation of new propositions in K ($C \rightarrow K$). The two spaces and four operators (including the $K \rightarrow K$) are shown in Fig. 2.

A space of concepts is necessarily tree structured as the only operations allowed are partitions and inclusions and the tree has an initial set of disjunctions. In addition, we need to distinguish between two types of partitions: restrictive and expansive partitions.

- If the property added to a concept is already known in K as a property of one of the entities concerned, we have a *restricting partition*;
- If the property added is not known in K as a property of one of the entities involved in the concept definition, we have an *expansive partition*.

In C–K theory, creative design occurs as a result of two operations: (1) using addition of new and existing concepts to expand knowledge; (2) using knowledge to generate expansive partitions of concepts.

4 Generating new entities and methods by infused design

Figure 1 shows the path that was employed for revealing a new concept in statics—the "face force" concept—from the ID perspective. The focus in the process is on the existing duality between trusses, from statics, and mechanisms from kinematics, shown in Fig. 3. Dual entity of entity X is denoted by X'. In this duality, the correlation between trusses and their dual mechanisms is as follows:

1. Geometrical relationship between the elements—for each bar in the truss, there is a corresponding link in the dual mechanism, drawn perpendicular to it.
2. Correlation between the external forces and the drivers—the external force applied to the truss is identical, both in direction and in magnitude, to the relative velocity of the driving link. As an example, in Fig. 3a, the external force P, represented as a vector, is identical to the relative linear velocity of the driver $V_{A/0}$ in Fig. 3b.
3. Topological correlation—every face, a circuit without inner edges, in the truss corresponds to a joint in the dual mechanism.
   For example, circuit B' defined by bars {2,3,4} in the truss corresponds to joint B in the dual mechanism.
4. Correlation between the elements—due to the above relationships between the truss and the dual mechanism, the forces in the bars are identical to the relative linear velocities of the corresponding links in the dual mechanism.

The process of revealing the new concept is comprised of several steps:

Step 1. The first step uses the "filling the hole in the map" heuristic.

The first step was the observation that when using duality between the PGR and FGR representations to transform mechanisms to determinate trusses, two basic concepts in mechanisms—*joint linear velocity* and *instant center*—do not have a corresponding entity in determinate trusses. These are the two missing holes that the approach revealed. The question is how rich is the available information that would allow filling them.

More specifically, the correspondence between trusses and mechanisms implies that for each entity or variable in one system, there exists an entity or variable in the other, shown in Table 1, which possesses the same value. The two missing entries in Table 1 could mean that we simply are not yet aware of these concepts or that we need to elaborate our knowledge with a richer modeling of the duality, that is, bring additional knowledge to bear on this issue from other types of dualities that exist in other representations. Before continuing, we name one of these holes by using the correspondence between the description of joint (face) and velocity (force). The hole corresponding to the instant center that is a more complex concept is named here as "equimomental line" as shown in Table 1. The origin of this concept is explained later.

Step 2. Now that we have revealed an unknown entity called "face force," designated by the letters FF, and defined solely by its duality with the joint linear velocity in the mechanism model, we want to investigate its nature and to learn about its attributes. In these two domains, statics and kinematics, such an entity does not exist; thus, a search for additional knowledge is gained in higher levels of representations that encompass more engineering disciplines. From Fig. 1 it can be concluded that RGR is a more general representation of the PGR and FGR and is

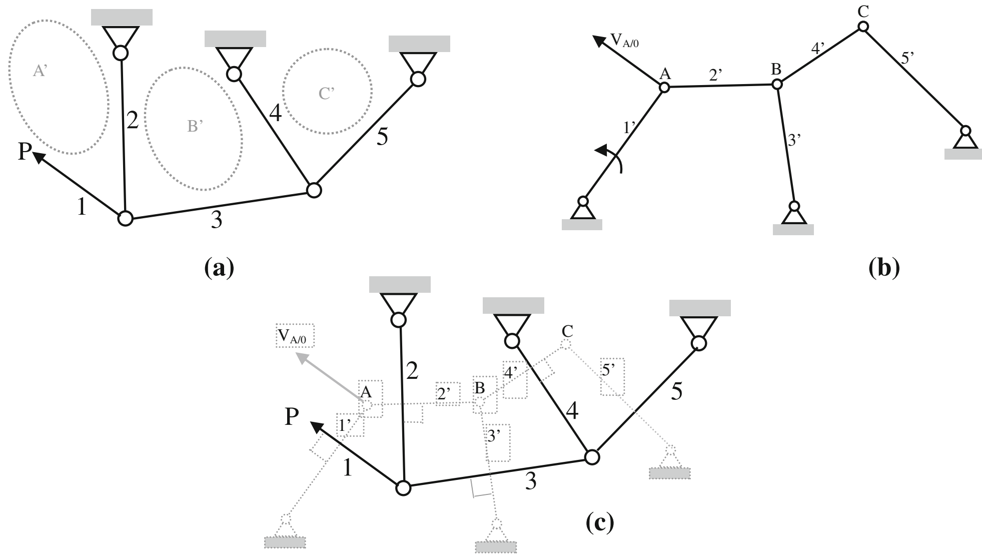

Fig. 3 The dualism between a truss and a mechanism. **a** The primal truss with the *indicated faces*. **b** The corresponding dual mechanism with a joint corresponding to each *truss face*. **c** The truss and the dual mechanism (*dashed line*) superimposed

Table 1 Duality between trusses and mechanisms: the duality relationship between these two engineering systems implies the complete correspondence between the variables describing them

Dual systems	Mechanisms	Determinate trusses
1	Relative velocity in a link	Force in a bar
2	Velocity	Force
3	Point (Joint)	Face (or contour of bars)
4	Joint linear velocity	*Unknown entity—face force*
5	Instant Center	*Unknown entity—equimomental line*

While the relative link velocity in the mechanism corresponds to the bar force in the truss, there is no existing variable which corresponds to the joint velocity

applicable also to electrical circuits. Once the knowledge that exists in electrical circuits is now available, it is possible to reveal, as can be seen in Table 2, that the face force in trusses is similar to mesh current in electrical networks.[2]

In electricity, there exists the following notion: The current in each circuit element is defined as the difference between the two mesh currents adjacent to it. For the sake of consistency, we subtract the left mesh current from the right one, as follows:

$$\text{Current (element } i) = \text{Mesh-current}_R - \text{Mesh-current}_L \tag{1}$$

where R and L stand for the right- and left-hand sides. For example, the current in resistor R3 in Fig. 4a is equal to the subtraction of the mesh current I from the mesh current II. Note, the mesh currents I and II are equal to the voltages of joints I and II, respectively, in the dual electric circuit shown in Fig. 4b.

In one-dimension systems, such as electrical circuits, we can assign a consistent value to the direction of a current across all the circuits, say, clockwise is positive. Now, according to Table 2, since both electrical current and force are of the same type, flows, we can formulate the following hypothesis:

$$\text{Force (element } i) = \text{Face-Force}_R - \text{Face Force}_L \tag{2}$$

Step 3. Since we are referring to the new entity as some kind of "force," we expect it to pertain to some line of application. In addition, we expect it to be related to other variables through quantitative equations that reflect the physics of the system. Let us summarize what can be concluded from Table 1:

1. The relative velocity of a link whose end joints are A and B (Figs. 3b, 5a) is the difference between their corresponding linear velocities, that is, $\overrightarrow{V}_{A/B} = \overrightarrow{V}_{A/0} - \overrightarrow{V}_{B/0}$ (Fig. 5b).
2. Let A' and B' be the faces in the dual graph corresponding to joints A and B (Fig. 3c, a). It was proved (Shai 2001a) that the force in the bar, $A'B'$, between these two faces (bar 2 in Fig. 3a) is equal to the relative velocity of the corresponding link between joints A and B (link AB) because these variables are dual (Table 1), that is, $\overrightarrow{F}_{A'B'} = \overrightarrow{V}_{A/B}$.

[2] Note that the potential difference and flows correspond to across and through variables used in control theory (Shearer et al. 1971).

Table 2 An extension of Table 1, since it includes also higher representation RGR, thus includes more engineering domains, such as electrical circuits

	Potential	Potential difference	Dual potential in the dual system	Dual potential difference in the dual system
Mechanisms	Joint linear velocity	Relative velocity of a link	Face force	Force in a bar
Trusses	Node displacement	Deformation of a bar	*Face force in the dual locked mechanism*	*Force in a link in the dual locked mechanism*
Electrical circuits	Voltage of a junction	Potential drop across an element	Mesh current	Currents in an element

We adopt this notation because we are using potentials in addition to potential differences and we are also using the duals of both. The *italicized entries* are given for completeness but are not used in this paper

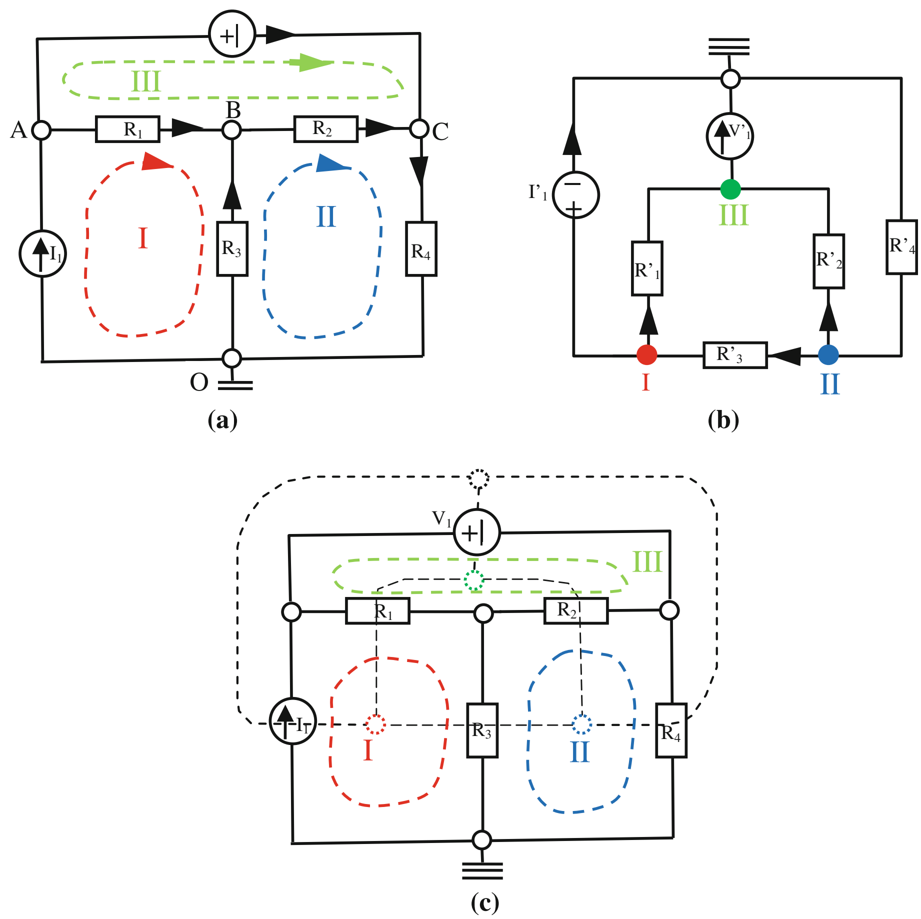

Fig. 4 The dualism in electrical circuits. **a** The primal circuit with the *indicated faces*. **b** The corresponding dual electrical circuit where a junction corresponds to a face in the primal circuit. **c** The primal electrical circuit and the dual circuit (*dashed line*) superimposed

3. Since the new variable, face force, is assumed to be equal to the linear velocity of the joint in the primal system, it follows that $\overrightarrow{V}_{A/0} = \overrightarrow{FF}_{A\prime}$ and $\overrightarrow{V}_{B/0} = \overrightarrow{FF}_{B\prime}$ (Fig. 5a, c).
4. From the above analysis, it follows that the force in a bar is equal to the difference between its two adjacent face forces; in the given example, $\overrightarrow{F}_{A\prime B\prime} = \overrightarrow{FF}_{A\prime} - \overrightarrow{FF}_{B\prime}$ (Fig. 5d).

In this step, we can claim that we have affirmed the existence of the face force and "discovered" how it relates to the forces in the bars. Alternatively, we can more precisely say that we have introduced a new entity that was not in the dual, a hole, and found its properties that could be deduced from the dual equations. Still, part of any force definition, its acting line, cannot be discerned from the duality relationship between the representations PGR and FGR. In addition, the last unknown entity in the dual, corresponding to the instant center in the primal, remains unknown. This situation is depicted in Table 3.

Reflection on this step: Naming an entity immediately constrains our perception of its further refinement. This is

Fig. 5 The relationship between the velocities of a link and the forces in the corresponding *dual bar*. **a**, **b** The absolute linear velocities of the end joints *A*, *B* define the relative linear velocity of the link. **c**, **d** The face forces adjacent to the dual bar define the force acting on it

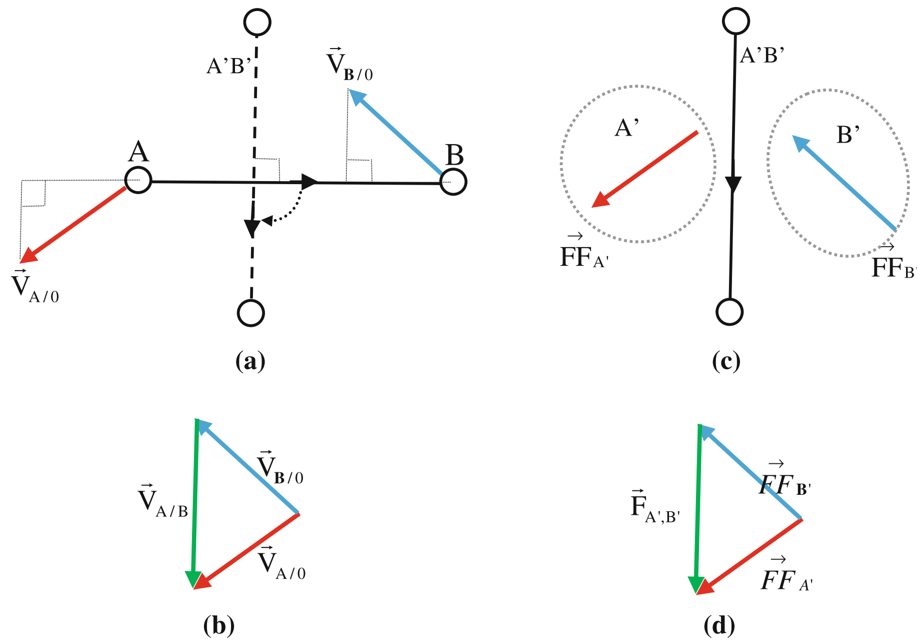

Table 3 An elaborated Table 1 showing the present understanding of the "holes" in the representations

Dual systems	Mechanisms	Determinate trusses
1	Relative velocity in a link	Force in a bar
2	Velocity	Force
3	Point (Joint)	Face (or contour of bars)
4	Joint linear velocity	*Face force*—**new entity**
5	Instant Center	**Unknown entity**—*equimomental line*
6	**Unknown entity**	*FF Acting line*—**new entity**

The *italic* denotes the names of new or unknown entities and the *boldface* states their status whether new or unknown

similar to design, where design progresses by coevolution of functions and structure. Such constraint may limit us but also provides a direction for further concept elaboration when the space of possibilities is large.

Step 4. An elaboration of the location of the face force to complete its definition requires revealing additional new knowledge. Till now, due to the duality between FGR and PGR, we revealed that there exists a missing entity in statics, a force that acts in faces. Since RGR, a representation applicable to statics is also applicable also to electricity; we revealed that the force is a variant of "mesh current." Since statics systems are known to be multidimensional, the next missing entity in statics is the line along which the face force acts. For this, we use one of the strengths of the IEKG—multiple representations, for example, the same engineering system can be represented by diverse representations, each is proper to deal with specific engineering properties. For example, mechanisms can be represented by LGR, PGR, and PLGR. Moreover, the PGR and FGR are representations more appropriate to deal with knowledge that exists in the elements; in statics, these would be forces in the bars, and in kinematics, the velocities of points in links. In contrast, PLGR and FLGR deal with knowledge related to the relationships between the elements.[3] Consequently, employing both representations might provide access to more knowledge as shown in the present discovery case.

Furthermore, abstract domains such as kinematics and statics are embedded within numerous specific domain systems, for example, statics within determinate trusses, pillar systems, indeterminate beams, and more. Since there are a number of representations associated with each domain, we have a new possibility to deal with the same systems and concepts from diverse perspectives. From the experience of using ID, including the present study, it becomes clear that there are many cases where knowledge is implicit in one representation and explicit in the other. This unique property is entirely different from other known methods used in the design community, such as bond graphs, where only one representation is used (Borutzky 2010).

As mentioned before, mechanisms can also be represented by PLGR, indicated in Fig. 1 by a dashed point line, which in turn, enables access to another representation—FLGR—through another duality relation. This new channel between kinematics and statics (implemented in pillar systems) exposes new knowledge that was not previously

[3] In 2-dimensional kinematics and statics, both representations could be used.

Table 4 Projective geometry duality between mechanisms and pillar structures

Dual systems	Mechanisms (PLGR)	Static Systems (FLGR)
1	Linear velocity	Moment
2	Angular velocity	Force
3	Point (joint)	Line

known, partly because the relationship between the representations is based, this time, on projective geometry. This second duality principle and its corresponding equivalences are shown in Table 4. The PLGR and FLGR representations and their duality enabled revealing the corresponding analogy of *the relative instant center in statics* (see Table 1), another new concept that was not known before.

Step 5. Let us investigate what can be concluded from basic text books in kinematics and statics about the instant center and its possible dual concept. Note that this analysis pertains to kinematics and statics in general and is not related specifically to the particular systems of mechanisms and determinate trusses.

1. Every link has a single point around which the link rotates. This point is called the absolute instant center.
2. The linear velocity of each point of the link due to its rotation can be calculated using the angular velocity and the distance between the point and the absolute instant center.
3. Every two links have a point where their absolute linear velocities are the same. This point is termed *relative instant center*.
4. The linear velocity of a link at the absolute instant center is equal to zero.

Relying on projective geometry duality (Table 4), angular and linear velocities correspond to force and moment, respectively, and point is transformed into a line. In kinematics, as indicated in statement 1, the motion of every link is characterized by its angular velocity, and there is a single point for each link where the linear velocity of the link is equal to zero.

In statics, although not defined previously, for each force, there is a line where the moment exerted by the force along this line is equal to zero. In this paper, we call this line the *absolute equimomental line*. Now that the transformation rule from kinematics into statics using projective geometry is given, the four kinematic statements above can be written in statics as follows:

1. Every force has a single line along which it acts. This line is called the absolute equimomental line.
2. The moment at each point in the plane due to the acting of a force can be calculated using the value of the force and the distance between the point and the absolute equimomental line.
3. Every two forces have a line where they exert the same moment. This line is termed the relative equimomental line.
4. The moment of a force along its absolute equimomental line is equal to zero.

These statements were subsequently used to focus the analysis between the dual systems.

The duality between angular velocity and force due to projective geometry is widely known in the literature and is used in screw theory (Davidson and Hunt 2004). Figure 6a, b illustrates the similarity between the field of velocities/moments in the plane due to the rotation/action of a link/face force around/along its absolute instant center/equimomental line.

Step 6. Now that the counterpart of the relative instant center in statics is known from step 1, we introduce the definition of the relative instant center as it appears in any kinematics textbook (Erdman and Sandor 1997).

Definition 1 The relative instant center of links x and y, $I_{x,y}$, is a point where the links, having angular velocities ω_x and ω_y, respectively, have the same linear velocity.

When we transform this definition into statics, we derive a new entity, eqm(x,y), defined as follows (maintaining the phrasing in the previous definition as much as possible):

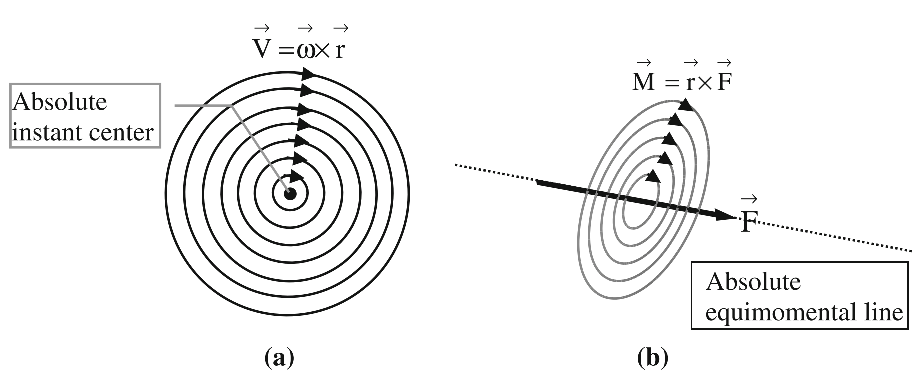

Fig. 6 The correspondence between: **a** the velocity field constructed by the link defined by its angular velocity, and **b** the force field constructed in a face and defined by its face force

Definition 2 The new concept, *equimomental line*—eqm(*x*,*y*) is a line where upon each point, the forces having values F_x and F_y exert the same moment.

or phrased differently,

Definition 3 The new entity, *equimomental line*—eqm(*x*,*y*), is a line where upon each one of its points, the forces F_x and F_y exert the same moment.

There is a complete correspondence between instant centers in kinematics and the equimomental line in statics, thus such correspondence should exist for any of their special cases. In kinematics, for instance, there is a special case of the instant center, called: *absolute instant center*, defined as follows (Erdman and Sandor 1997):

Definition 4 *Absolute instant center*, $I_{x,o}$,is a point in which the linear velocity of link x is equal to zero.

Transforming this special entity into statics yields:

Definition 5 *Absolute equimomental line*, *eqm(x,0)*, is a line in which the moment exerted by force x is equal to zero.

From the physical point of view, this is the line where the force acts; thus, along this acting line, it exerts a zero moment.

Step 7. Till now, we have seen the transformation of variables from kinematics into statics. Now, we will show that the transformation through the duality also enables us to derive new theorems in statics from kinematics. Let us transform the known Kennedy theorem in kinematics into statics yielding a new theorem in statics.

Kennedy Theorem Suppose we have three links, x, y, and z; it follows that the three relative instant centers, $I_{x,y}$, $I_{y,z}$, and $I_{x,z}$, are collinear. Applying the dual projective geometry to the Kennedy theorem and using the duality relation that maps collinear points into lines that all intersect at the same point yields the following new theorem:

Dual Kennedy theorem in statics Suppose we have three forces: F_x, F_y, and F_z; it follows that the three relative equimomental lines eqm(*x*,*y*), eqm(*y*,*z*), and eqm(*x*,*z*) intersect at the same point.

Now, we are ready to refer to the original question of the location where the face force acts. Following the definition of the equimomental line, every force, in particular the face force, acts along the absolute equimomental line. Now, we are faced with a need to come up with an algorithm to find the needed equimomental lines. Following the idea introduced in this paper, we transfer the problem to kinematics where there exists a known method, Kennedy Circuit method, for finding all the instant centers—the dual to the equimomental lines. Next, we need to transform the method from kinematics into statics, yielding an algorithm for finding all the equimomental lines, as appears in (Shai and Pennock 2006).

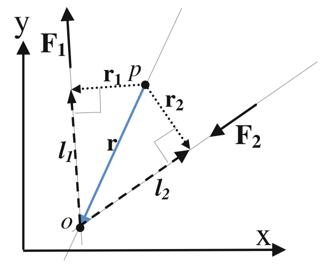

Fig. 7 Geometrical description of the equimomental line

Proposition 6 *From the physical point of view, the equimomental line of two forces is a line defined by the vector difference between these two forces.*

Proof The equimomental line of two forces is a line where the moments exerted by these two forces along each point on the line are the same. This property can be written as follows:

$$\mathbf{r}_1 \times \mathbf{F}_1 = \mathbf{r}_2 \times \mathbf{F}_2 \tag{3}$$

Any two lines in the plane always have a crossing point. Let us designate this point by o. The two forces exert the same moment at the crossing point—zero moment. Thus, the equimomental line should pass through this crossing point.

Let us choose an arbitrary point along the equimomental line (Fig. 7) designates by p. We will define $\mathbf{r}$ as the radius vector from p to the crossing point—o, that is, $\overrightarrow{r} = \langle p, o \rangle$. The moment exerted by the two forces at point p is:

$$\begin{aligned} (\overrightarrow{\mathbf{r}} + l_1\hat{e}_1) \times (F_1\hat{e}_1) &= (\overrightarrow{\mathbf{r}} + l_2\hat{e}_2) \times (F_2\hat{e}_2) \\ \overrightarrow{\mathbf{r}} \times (F_1\hat{e}_1 - F_2\hat{e}_2) &= 0 \end{aligned} \tag{4}$$

which means that the direction of the equimomental line is the same as the direction of the vector difference of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$.

4.1 Example of using the new face force concept

Now that we have a new concept in statics, it is reasonable to expect that it would lead to new practical applications. This section demonstrates the use of the face force concept in a specific type of structures, termed "tensegrity structures," which consist of strut and cable elements, sustaining only compression and tensile forces, respectively.

In tensegrity structures, there should always be an internal force, termed self-stress, maintained by compression in the struts and tension in the cables. The struts also work as actuators enabling the structure to adjust its configuration. Therefore, in these types of systems, there is a need to know and control the forces in all the elements in order to prevent a situation where the system collapses.

In kinematics, the fundamental property underlying mechanisms is that once the angular velocity of the driving link is known and all instant centers are found, all the velocities of each joint and link can be determined. The same fundamental property exists in statics. Once one of the face forces is known and all the equimomental lines found, all the forces in the elements can be determined as a function of the given face force.

This idea will be demonstrated through the structure given in Fig. 8. In Proposition 6, it was mathematically proven that the equimomental line of two forces lies along their difference vector. The outer face, designated by the number zero, will be termed the *reference face*, and the face force acting in that face is set to zero. It follows from Proposition 6 that the absolute equimomental lines of the faces adjacent to the reference face are along the outer bars in the outer contour, that is, those bars that are adjacent to the reference face. For example, in Fig. 8b, it is shown that the absolute equimomental lines of the faces I, II, III, and V are along the outer bars 1, 2, 3, and 4, respectively.

Now, to find the absolute equimomental lines of the inner face, which is not adjacent to the reference face, we will have to apply the dual Kennedy theorem. We clarify this procedure by showing how to find the equimomental line of face IV, that is, eqm (IV,0). Let us take the three faces 0, I, and IV. From the dual Kennedy theorem, it follows that the three relative eqms (0,I), (I,IV), and (IV,0) should intersect at the same point. Since the eqms of the first two are along bars 1 and 8, intersecting at point *A* (Fig. 8b), it follows that the eqm (IV,0) should pass through point *A*. Now, let us again apply the dual Kennedy theorem on the faces IV, III, and 0, resulting in that the eqm (IV,0) should also pass through point *B*. Now that we have the two points through which eqm (IV,0) must pass, the eqm (IV,0) is defined as shown in Fig. 8b. Now, we know the incline line along which the face force acts (IV,0), but we need to know its direction. For this, we use the fact that bar 8 is adjacent to both faces I and IV, which means that it is the eqm (I,IV). In other words, the moments exerted by the two face forces (I,0) and (IV,0) about any point along this bar should be the same. Let us choose a point, designated by C, as shown in Fig. 8c. Since the direction of moment exerted by face force (I,0) is clockwise, the face force (IV,0) should exert a clockwise moment defining its direction. Applying the same idea allows us to determine the directions of all the face forces (Fig. 8d), which define the type of element, cable, or strut, as shown in Fig. 8e.

We now conclude this exercise in ID, having discovered two new entities with their associated meaning and valuable methods. Such discovery has not taken place till now and the authors are not aware of any result reported in the literature that using a systematic method led to such discovery. Furthermore, even the presentation of these new entities and their meaning (without describing the way they were discovered) led to enthusiastic response in the mechanical engineering community. Establishing the “face force” variable is not only important from the theoretical point of view, but has practical analysis applications as well (Shai 2002).

The discovery was made possible by the formal representations used to model diverse disciplines. These representations allow to locate holes and to bring the necessary required information from using other representations in the map of representations, which we recently named the *IEKG* (Reich and Shai 2012).

More specifically, the idea of using different representations provides us with a new view on work done in different disciplinary communities, thus enabling to infuse and create new concepts, theorems, and methods relying on the knowledge from different disciplines.

This turns out to be a powerful approach that extends beyond this study. For example, ongoing research is being performed now to investigate the kinematical analysis methods done in mechanical engineering using Jacobian matrices, and the kinematic methods in biology done relying on the properties of rigidity matrices. Preliminary results indicate that the former uses PLGR representations and the latter PGR representations (Slavutin et al. 2012). Elaborating the two methods indicates that there is an opportunity of creating a new method that will capture the advantages of the methods used by two different communities.

5 Modeling the “face force” discovery with a C–K perspective

We now show how C–K can model the previously described ID discovery process. A complete general analysis requires a separate study, but on the example of the truss–mechanisms duality, we can highlight several remarks that help to unveil the design logic of scientific discovery. The present analysis follows the 7 steps of the discovery process.

Step 1: The first step where the unknown concept appears could be understood as:

Using the ID framework: Analysis of K reveals omission of knowledge k_m: the duality between trusses and mechanisms shows *unknown entities* related to trusses. The dual provides a basis for a specific K–K operator that allows the identification of potentially missing knowledge. The missing knowledge k_m is not understood in its “neighborhood” (discipline); it has no identifiable meaning in existing practice.

k_m is used to generate a new concept called Face Force, that is, a force that is intentionally designated as the dual of

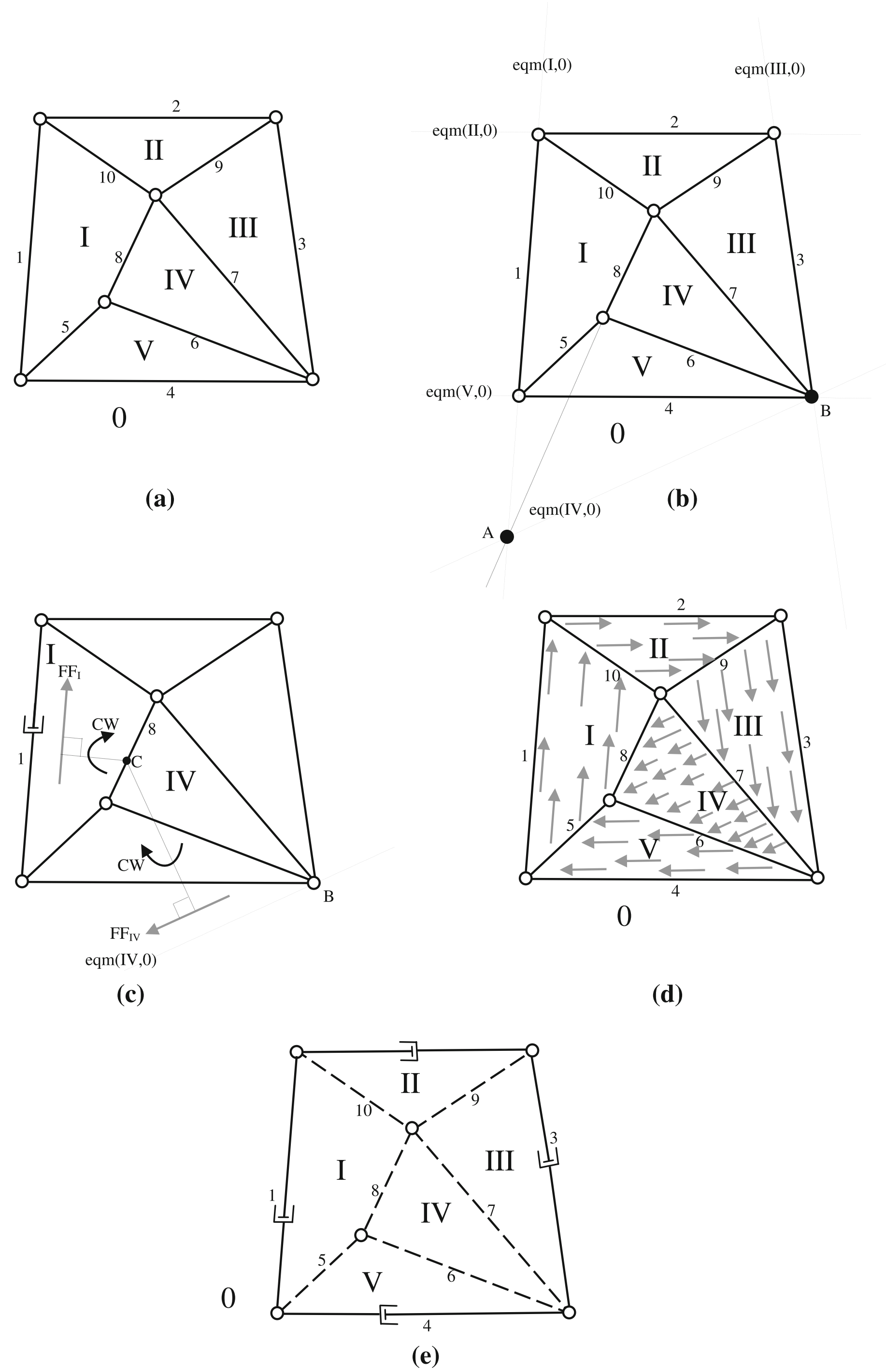

Fig. 8 Example situating the elements of a tensegrity structure through the face force method. **a** Initial structure. **b** All the absolute equimomental lines. **c** Determining the direction of the face force in face *IV*. **d** The map of all the face forces and their directions. **e** The resultant rigid tensegrity structures

joint velocity [$K \rightarrow C$]. This is clearly an expansive partition in K of "face" or "force" (there is no such thing in K as a "face force") and the complete definition of a Face Force is undecidable at the beginning of the process.

Remark I: Unknown holes as potential concepts. The embedded scientific knowledge in the duality equations is used not only to map the corresponding entities from the primal to the dual of what is known but also to reveal what is unknown. It is also worth mentioning that the knowledge k_m that would define the Face Force cannot be deduced from previously existing knowledge. The "face force" is neither induced or is the result of an abduction process as there are no "face forces" that have been observed. It is neither a hypothesis, in the classic scientific sense: it is an unknown notion whose existence cannot be tested as it is yet undefined. This is the most intriguing step. At this stage, the face force is neither "discovered" like we would say of "a new planet" nor "invented" as we do not know what it is.

The "face force" is a concept as defined by C–K theory and its design can begin. And the "filling the hole in a map" heuristic is a basic operation of the ontology of Design (Hatchuel et al. 2013).

Step 2: Once in C, different knowledge sources are activated to expand and understand the concept of the "face force" with new attributes (partitions in the C–K language). Past studies (i.e., previously generated knowledge) help elaborate the concept, so it is related to the force in bars and to mesh currents [$C \rightarrow K$].

Step 3: Relevant knowledge is used to identify further unknown attributes t [$K \rightarrow K$]. Through another representation, pillar systems become interesting candidate source of knowledge.

Remark II: Knowledge consistency. These knowledge expansions are guaranteed to generate knowledge without contradictions, without the need to check this property. Otherwise, knowledge elaboration might lead to contradictions that are impossible to remove automatically.

Step 4: Additional knowledge is gathered by further elaboration of the mechanism–pillar system relation [$K \rightarrow K$]. It becomes clear that there is useful knowledge in this domain to elaborate the face force concept.

Step 5: This step further elaborates on the projective geometry duality between statics and kinematics by outlining the notion of absolute equimomental line [$K \rightarrow K$].

Remark III: A new entity is introduced but it is not an unknown; it is a known entity that was not named but will serve to expand the face force concept. It is interesting to note that the introduction of an unknown object triggers operations of *K*-reorderings (Hatchuel et al. 2013). This reordering is not a creation of new knowledge but it prepares the following expansions of the concept.

Step 6: The concept of absolute equimomental line is attributed to the face force [$K \rightarrow C$]. And this operation provides the final missing part in completing the definition of the face force concept. This step moves the concept into the K space [$C \rightarrow K$].

Step 7: This step continues the *K*-reordering operations induced by the duality theorems and further elaborates on the equimomental lines of two forces in statics.

Remark IV: The necessity of K-Reorderings. From a C–K perspective, discovery occurs when a new concept is formed and subsequently transferred to K. A jump from K to K in a way that creates a new entity is impossible. The C–K representation map in Fig. 2 seems to support $K \rightarrow K'$ "macro" operators (like duality) where knowledge from one discipline K is transferred to knowledge in another discipline K′. This K-reordering process is necessary but not enough. It allows finding "holes," and it also derives all consequences of the new entity that has been designed. However, new entities need the formation of complete new paths of the type $K \rightarrow C \rightarrow K'$ that come across new concepts.

Remark V: How ID triggers creativity in design. As can be seen above (and from Shai et al. 2009a), the expansion of K by new knowledge is done in a systematic way by using additional representations of mechanisms and trusses. Usually, the expansion of K by new knowledge is derived from experiments or other external sources. ID opens a new way for expanding K by augmenting existing disciplinary K with other, seemingly unconnected, knowledge sources. Yet, ID serves as a new bridge to connect pieces of seemingly disparate knowledge in a consistent manner, so that they could be brought into C for generating new concepts. Initially, these new concepts do not have understandable meaning in the discipline of the dual, although some elements of their definitions (assured existence with some meaning) are guaranteed by ID operators. Thus, the interpretation of ID with C–K theory throws an interesting light on scientific discovery, which in this case is clearly a design process (i.e., it needs a C-space); however, the K-expansion is controlled by special $K \rightarrow K$ operators that warrant consistency and compatibility: the new objects have to obey to pre-established knowledge and these rules warrant some aspects of their existence. Clearly, the face force is still a force in the classic sense even if its formation and action line are unique.

6 Discussion: scientific logic as creative design

This study has several contributions to design theory, practice, and science in general.

6.1 Creativity in science as a design process

In relation to design theory, we need to ask "Do I know more on C–K or ID from the analysis?" First, in the context of our study of creativity theories and methods, we find that C–K could model the particular ID process presented in this paper. In addition to a previous study (Reich et al. 2012), this strengthens the claim that C–K as a design theory embeds creativity as an inherent part. But the study offers deeper insight as C–K theory was applied primarily to model the creative design of engineered artifacts, not scientific results (Hatchuel and Weil 2003, 2009). The study of ID offers a direct opportunity to extend C–K theory towards the generation of symbolic artifacts like scientific entities. The previously established correspondence between C–K theory and forcing in modern set theory (Hatchuel and Weil 2007; Hatchuel et al. 2013) opened this line of development, as Forcing is a method to design new infinite sets (extension models for Set theory), which are also scientific objects. A simpler example of design as a scientific logic can be also found in the formation of complex numbers through the introduction of the new concept of $i = \sqrt{-1}$. It may be interesting to remark that the introduction of such complex numbers comes from a duality principle between the real and the imaginary roots of the polynomial functions (Van der Waerden 1985). As is the case with the concept of "face force" in this paper, most roots of polynomial equations were unknown in the world of the real numbers. The design of complex numbers as a well-defined number field created a new source of knowledge that was consistent with real numbers and could generate new roots for any polynomial function.

In this paper, we cannot do complete justice to the conjecture that creativity in science through the creation of symbolic artifacts is a design process. ID and other examples are convincing enough to open a new research program on C–K theory as a model for analyzing and interpreting the logic of scientific discovery.

6.2 Knowledge-based systems with embedded C–K logic: a new type of design support systems

First, ID raises a new issue in knowledge management within C–K or any other theory or practical design support system. Managing a knowledge base, including its consistency is a challenging task. Operators that extend knowledge and maintain its consistency could be extremely powerful.

Second, intricate knowledge bases include significant hidden insight that could be discovered, leading to the formation of new concepts and potentially after further expansions to new surprising knowledge. Operators that create such new concepts, first operate from K to K trying to identify disjunctions, "holes" or interesting unknown objects. This requires a revision, re-interpretation or enrichment of $K \rightarrow K$ operators. As such, ID is both a special organization of engineering knowledge and a design support method.

As a scientific knowledge, ID provides an interesting multilevel structure.

Level 1 Graph theory (or matroid) is the highest level and the least specific form of knowledge.

Level 2 Flow graphs and Potential graphs are not implications of graph theory, but its combination with specific algebras (flows, potential, or even durations in transport problems), which are added to the graph structure. Truss–mechanism duality appears at this level as shown in Fig. 2.

Level 3 Engineering specialties are at the lowest level; they also introduce new additional knowledge to reach some sort of embodied form of knowledge (materials, fluids, energy…), which appear as isolated domains.

The classic logics of engineering design and computation tend to favor a design process that stays at the embodiment level of this structure where solutions seem "realistic," "concrete," or testable. ID allows to avoid such "embodiment trap:" it offers to travel horizontally, at level 2, in Space K, yet with a rigorous and controlled sets of operations. Actually, analogies and metaphors are well-known sources of creativity through jumping from one domain to another. They may generate new concepts, but without any consistent source of K or method for K-expansion that could provide the progressive elaboration of these concepts. ID avoids such potentially misleading and useless generation of concepts; it helps to think out of the engineering boxes, in a controlled and rigorous manner. The concepts generated through duality can be clearly designed at the intermediate level of the graph algebras. They also could offer an important design support at the embodiment level if there is at least one tractable solution in one of the embodiment domain. If the latter exists, its dual might be identified and it might serve as consistent design candidate, that is, concept that is close to be perceived by experts as a solution. Sometimes, identifying a solution at the embodiment level might not be easy (Shai et al. 2009a).

While ID has been shown to support creative design and create new scientific knowledge, its interpretation with C–K theory helped us to identify the point where the creative act occurs. More generally, ID may be a special case of a new type of knowledge-based methods of systems (or meta knowledge-based systems) that possesses an embedded C–K logic inside; another approach is explored by Kazakci et al. (2008). The general and complete characterization of this type of hybrid structures between scientific consistency and design logic is yet to be explored and will be pursued in future research.

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